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The inverse problem of determining the filtration function and permeability reduction in flow of water with particles in porous media

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Abstract Deep bed filtration of particle suspensions in porous media occurs during water injection into oil reservoirs, drilling fluid invasion of reservoir production zones, fines migration in oil fields, industrial filtering, bacteria, viruses or contaminants transport in groundwater etc. The basic features of the process are particle capture by the porous medium and consequent permeability reduction. Models for deep bed filtration contain two quantities that represent rock and fluid properties: the filtration function, which is the fraction of particles captured per unit particle path length, and formation damage function, which is the ratio between reduced and initial permeabilities. These quantities cannot be measured directly in the laboratory or in the field; therefore, they must be calculated indirectly by solving inverse problems. The practical petroleum and environmental engineering purpose is to predict injectivity loss and particle penetration depth around wells. Reliable prediction requires precise knowledge of these two coefficients. In this work we determine these quantities from pressure drop and effluent concentration histories measured in one-dimensional laboratory experiments. The recovery method consists of optimizing deviation functionals in appropriate subdomains; if necessary, a Tikhonov regularization term is added to the functional. The filtration function is recovered by optimizing a non-linear functional with box constraints; this functional involves the effluent concentration history. The permeability reduction is recovered likewise, taking into account the filtration function already found, and the functional involves the pressure drop history. In both cases, the functionals are derived from least square formulations of the deviation between experimental data and quantities predicted by the model.

Keywords Deep bed filtration · Suspension transport · Porous media · Inverse problem · Tikhonov regularization · Formation damage · System of convection-reaction equations

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1 Introduction

Severe injectivity decline during sea—or produced water injection is a serious problem in offshore waterflood projects. This decline results from permeability impairment because the rock captures particles from the injected water. Reliable modelling-based prediction of injectivity decline is important for the design of injected water treatment or management by water filtering, injection of sea—and produced water combinations etc.

During flow of water with suspended particles through porous media, the particles are gradually retained, reducing the permeability of the medium. This phenomenon is called *deep bed filtration with formation damage*. Formation damage can be induced by the penetration of drilling fluid into a reservoir. Other petroleum related applications for which filtration and formation damage are important include sand production control, fines migration, disposal of produced water in aquifers and deep bed filtration in gravel packs. Particle suspension filtration also occurs in industrial water filtering, in propagation through aquifers of contaminants (including viruses, bacteria etc.), and in other environmental processes.

Mathematical models for suspension transport in porous media consist of an advection-diffusion equation of particle mass balance with a particle capture term and Darcy's law accounting for permeability reduction due to particle retention (see Elimelech et al. 1995; Herzig et al. 1970; Kuhnen et al. 2000; Pang and Sharma 1994; Payatakes et al. 1974; Sharma and Yortsos 1987 and Tufenkji et al. 2003). The models contain two empirical functions describing properties of the aqueous suspension and of the porous medium: the filtration function i.e. the probability for a particle to be captured per unit particle path length, and the permeability reduction i.e. the ratio between the reduced and the initial permeabilities. Particle deposition alters pore space geometry and hydraulic resistivity; in turn, pore geometry alters conditions for further deposition, so it is natural to take the deposited concentration as the basic independent variable for the filtration and formation damage functions.

Laboratory coreflood tests with particle suspension are carried out routinely in order to estimate injectivity decline during water injection into oil reservoirs (see Pang and Sharma 1994 and Wennberg and Sharma 1997). In these one-dimensional experiments, the retained profile is non-uniform: in the laboratory, it is impossible to create a suspended-retained particle system with uniform deposited concentration. Therefore, the filtration and formation damage functions cannot be measured directly. However, it is possible to measure the time series of suspended particle effluent concentration and of pressure drop, and these two functions can be recovered indirectly from these experimental measurements by solving inverse problems for deep bed filtration and formation damage.

For modelling purposes, the filtration and damage functions are given suitable parametrizations that are compatible with the physical properties of the phenomenon. The filtration function reduces to a constant coefficient for the case of diluted suspensions; it can be calculated from the mean effluent concentration. The formation damage function is usually represented by a hyperbolic formula containing one empirical parameter—the so-called formation damage coefficient—which can be calculated from the pressure drop along the core (Herzig et al. 1970; Pang and Sharma 1994; Wennberg and Sharma 1997). Alternatively, both coefficients can be calculated from pressure data at three core points (Bedrikovetsky et al. 2001, 2003).

In this work, we present a more flexible method for determining the filtration and formation damage with any arbitrary form using parameter optimization. Differently from other methods, these quantities are not assumed to be constant, rather they are functions of the deposited particle concentration. We introduce regularization to generalize the procedure given in Al-Abduwani et al. (2004). The recovery procedure for the filtration and permeability reduction functions presented in this work consists of optimizing certain functionals using the projection gradient method with box constraints developed in Birgin et al. (2000). The functionals to be minimized are obtained from a least squares formulation taking into account the difference between experimental data and quantities predicted by the model. The box constraints reflect physical properties of the solution such as positivity and monotonicity. The functions obtained from the coreflood data may then be used for predicting well injectivity decline during water injection by solving the direct problem for deep bed filtration.

This article is organized as follows. In Sect. 2, we present the dispersion-free deep bed filtration model with formation damage as a system of two hyperbolic equations for suspended and retained concentrations and a pressure balance equation. We also explain how to solve this system by integrating two families of ordinary differential equations. In Sect. 3, we describe the optimization procedures which we use to solve the two inverse problems. In the first one, we use the calculated effluent concentration history to define the functional that is minimized to determine the filtration function. In the second one, we use the deposition calculated using this recovered filtration function to predict the pressure drop history and we define the functional that is minimized to determine the permeability reduction function. In Sect. 4, we validate the recovery methods and we briefly examine the sensitivity of these inverse problems by means of synthetic data. In Sect. 5, we apply the method to experimental data and discuss the recovered functions as obtained by solving the two inverse problems.

2 Flow of water with particles in porous media

In this section we present the physical model for the flow of water with suspended particles suffering retention in porous media. This model was developed in Bedrikovetsky et al. (2001) based on Herzig et al. (1970). Neglecting particle diffusion and dispersion, the mass balance equation for linear flow accounting for suspended and retained particles is

$$\frac{\partial}{\partial t}(\phi c + \sigma) + U\frac{\partial c}{\partial x} = 0. \tag{2.1}$$

Here the volumetric concentrations of suspended and deposited particles are respectively $c(x,t) \in [0, 1]$ and $\sigma(x,t) \in [0, \phi]$. The dimensionless quantity $\phi \in [0, 1]$ is called the *rock porosity*: it is the fraction of the rock volume available to the fluid. It is further assumed that the overall porous space is available for small particles, that size exclusion is not important, and that one particle plugs one pore. In the cases we study $\phi \approx 0.2$, while c is of the order of 10^{-6} and the number of pore volumes injected is of the order of 10^3: the deposition observed is at most one order of magnitude above that of the injected concentration, therefore $\sigma \ll \phi$ and we can assume that ϕ is constant.

We consider the flow of diluted suspensions, where the "particle-pore reaction" can be assumed to be of order one for small suspended concentrations i.e. the retention rate is proportional to the suspended concentration c:

$$\frac{\partial \sigma}{\partial t} = \lambda(\sigma)Uc. \tag{2.2}$$

The dependence of the retention rate on σ is expressed by $\lambda(\sigma)$, which is called the *filtration function*: it is the probability for a particle to be captured per unit length of the trajectory (Herzig et al. 1970; Shapiro et al. 2007). Equation 2.2 implies that the retention rate $\partial\sigma/\partial t$ is assumed to be linearly related to the flow velocity U (Herzig et al. 1970; Iwasaki 1937).

This follows from the fact that the quantity of particles available for retention is proportional to Uc, see remark 2.1.

The filtration function is assumed to be independent of the retained particle concentration i.e. constant in σ, for most laboratory tests on bacteria transport, where a diluted colloid flows during a short time corresponding to just a few injected pore volumes (Elimelech et al. 1995; Logan 2001). In this case, the retained concentration is low, and particle deposition does not alter the rock surface. In the case of longer flow periods or higher injected concentrations, we assume a Langmuir retention kinetics where the retention rate is proportional to the number of "vacancies," so that λ depends linearly on σ (the so-called blocking function, see Johnson and Elimelech (1995) and Kuhnen et al. (2000)). In contrast, the random sequential adsorption (RSA) approach employs a filtration function that is a non-linear function of σ (also in Johnson and Elimelech 1995).

During seawater flooding of oil reservoirs, a large amount of water is injected, corresponding to millions of times the volume of the damaged zone around the well, and the retained particles may occupy up to a 5–30% fraction of the porous volume near the well. Therefore, the structure of the pore space and surface where particles are retained changes completely during injection (Pang and Sharma 1994; Roque et al. 1995; Veerapen et al. 2001; Wennberg and Sharma 1997), and the function $\lambda(\sigma)$ can become non-linear and even non-monotonic.

Remark 2.1 The retention rate in Eq. 2.2 can be interpreted in different ways. Several authors drop the velocity factor (Corapcioglu and Choi 1996; Logan 2001), and λ becomes the probability for particles to be captured per time unit. In this case, λ is a function of velocity expressed in terms of dimensionless parameters. Nevertheless, in several important cases the form of this function suggests that λ is proportional to U (Logan et al. 1995; Rajagopalan and Tien 1976), and the retention kinetics equation becomes (2.2). In Herzig et al. (1970) and Altoe et al. (2006), it is assumed that the capture rate by "vacancies" is proportional to particle flux. In the dispersion-free case the flux is equal to Uc, which also leads to (2.2).

We assume that the permeability reduction expressed by $k(\sigma)$ is due to particle retention, and that it is a decreasing function of the retained concentration (Pang and Sharma 1994; Wennberg and Sharma 1997). The "momentum balance" equation has the form of Darcy's law relating the flow rate U to the pressure p:

$$U = -\frac{k_0\,k(\sigma)}{\mu}\,\frac{\partial p}{\partial x}. \tag{2.3}$$

Here, k_0 is the absolute rock permeability and $k(\sigma)$ is the permeability reduction due to the retained particles σ. When expressed as a function of σ, it is called the *formation damage function*. It is normalized so that $k(0) = 1$ i.e. it is one for clean porous rock. In general, the water viscosity μ can be considered constant for small suspended particle concentrations.

Equation 2.3 implies a relation between the flow velocity U and the pressure drop along the porous medium Δp, which can be obtained integrating this equation in x. When conducting experiments, the flow is controlled either by specifying the pressure drop or the injection rate (and thus the velocity): the other quantity varies in time according to Eq. 2.3. In this work we analyse experiments conducted with specified flow rate i.e. with constant U.

Equations 2.1–2.3 form a closed system of three equations for there unknowns—c, σ and p. It is assumed that the filtration function is independent of the pressure p: therefore, the first two equations decouple from the third and form a system for the two unknowns c and σ. The physical domain is $t > 0$ and $0 < x < L$, where L is the length of the core.

Although the classical colloid filtration theory is widely used, several cases where the model (2.1)–(2.3) was unable to adjust the experimental data have been presented in the literature (Tufenkji and Elimelech 2004; Bradford et al. 2002). Even when breakthrough curves were well adjusted by the model, high deviations from measured data were observed in the simulated retention profiles. Conversely, high deviations between measured and modelled breakthrough curves were observed when the model was tuned based on retention profiles.

Remark 2.2 It is worth mentioning that the single capture mechanism expressed in system (2.1)–(2.2) may also be used to describe deep bed filtration with several simultaneous suspended particle capture mechanisms, such as size exclusion, attachment and gravity segregation (Guedes et al. 2006). Some network models for deep bed filtration consider the situation where several particles must be deposited in a single pore until its complete plugging (Siqueira et al. 2003). In this case, all intermediate interactions "suspended particle—partly plugged pore" can also be aggregated into a single capture kinetics described by Eq. 2.2, see Shapiro et al. (2007). The system (2.1)–(2.2) can be obtained by averaging micro-scale stochastic equations for the size exclusion and the attachment mechanisms of particle retention (Sharma and Yortsos 1987; Shapiro et al. 2007). For highly concentrated suspensions, the capture mechanism of bridging becomes dominant; the order of the "particle–pore reaction" may be much larger than one (Al-Abduwani 2005). In this case, the retention rate is not proportional to suspended concentration anymore. Modelling these processes is outside the of scope of the current article.

2.1 Boundary and measured data

As initial data, we assume that the rock is clean and contains water with no particles; as boundary data, we assume that the solid particle concentration entering the porous medium is given:

$$\sigma(x,0) = 0 \quad \text{and} \quad c(x,0) = 0, \tag{2.4}$$

$$c(0,t) = c_i(t) > 0, \quad t > 0. \tag{2.5}$$

The quantity $\sigma(0,t)$, however, needs to be determined utilizing the model. Along the line $x = 0$, we obtain from Eqs. 2.2 and 2.5:

$$\frac{\mathrm{d}\sigma(0,t)}{\mathrm{d}t} = \lambda(\sigma(0,t))Uc_i(t), \quad \text{and} \quad \sigma(0,0) = 0, \tag{2.6}$$

where U is given. Integrating Eq. 2.6 provides $\sigma(0,t)$, which is positive and increasing. In laboratory experiments, measurements of the pressure head $\Delta p_{\mathrm{exp}} = p(0,t) - p(L,t) > 0$ and effluent concentration $c(L,t) = c_{\mathrm{exp}}(t) > 0$ are also conducted: these data are used as input for inverse problems.

2.2 Solution for suspension flow

The well-posedness of the boundary/initial value problem (2.1)–(2.2) with boundary and initial data (2.4)–(2.5) was established in Alvarez et al. (2006) and Alvarez (2005), where U was taken as a constant and it was assumed that the filtration function $\lambda(\sigma)$ was C^1 i.e. it had one continuous derivative, and that $\lambda(\sigma) > 0$ for $\sigma \in [0, \phi]$. A generalization of this result follows. For $t \le (\phi/U)x$, $\sigma(x,t)$ and $c(x,t)$ vanish. For $t > (\phi/U)x$, one can rewrite Eqs. 2.1–2.2 on characteristic lines $x - (U/\phi)t$ =const in the form

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x} = -\lambda(\sigma)\sigma, \tag{2.7}$$

$$\frac{\mathrm{d}c}{\mathrm{d}x} = -\lambda(\sigma)cU \tag{2.8}$$

where d/dx means differentiation along characteristic lines $x-(U/\phi)t$ =const (Alvarez et al. 2006). Since $c(0,t) = c_i(t)$ is specified and $\sigma(0,t)$ is obtained from solving Eq. 2.6, the family of Eqs. (2.7)–(2.8) can be solved numerically using standard procedures for ODE's.

Although the non-linear inverse problem is inherently ill-posed, the following remark reflects that the direct problem is well-posed. This will be useful for proving that the regularized approximation of the inverse problem is well-posed, in the sense of Tikhonov, in some appropriate compact set (see Appendix A).

Remark 2.3 Since the solution σ and c of the system (2.7) and (2.8) are given by an ordinary differential equations along characteristic lines, the continuity of the solution is a consequence of the theorem on continuity of ODE solutions with respect to parameter changes (see Hille 1969, p. 91). So the maps

$$\lambda(\sigma;\theta) \to \sigma(x,t;\theta), \quad \lambda(\sigma;\theta) \to c(x,t;\theta) \tag{2.9}$$

are continuous in the uniform norm.

Differently from Alvarez et al. (2006), where $\lambda(\sigma)$ was strictly positive, we assume that $\lambda(\sigma)$ is a non-negative piecewise C^1 function that may vanish in its domain $0 < \sigma < \phi$ i.e. $\lambda(\sigma) > 0$ for $0 \le \sigma < \sigma_0$ and $\lambda(\sigma) = 0$ for $\sigma_0 \le \sigma < \phi$. The following Lemma, proved in Alvarez (2005), allows this assumption.

Lemma 2.4 *The solution of (2.1)–(2.3) with data (2.4)–(2.5) is given by (2.7)–(2.8) in the trapezoidal domain* $0 \le x \le L$, $0 < t \le (\phi/U)x + \tau$; τ *can be infinite. If it is finite,* σ *is constant and equal to* σ_0 *in the trapezoid* $0 \le (\phi/U)x + \tau \le t$ *and* $c(x,t) = c_i(x-(U/\phi)t)$. *Also,* c *is continuous in the trapezoid* $0 \le x \le L$, $t > (\phi/U)x$, *and* σ *is continuous in the infinite rectangle* $0 \le x \le L$, $0 < t < \infty$.

In Alvarez et al. (2006) and Alvarez (2005) the well-posedness of the direct problem was proved. To solve the inverse problems for the filtration function $\lambda(\sigma)$ and the formation damage function $k(\sigma)$, we need to solve Eqs. 2.1–2.5 and 2.6 many times as part of an iterative optimization procedure. Thus, it is necessary to solve this system with high speed and accuracy.

3 Recovery methods

In this section, the values of the coefficients that parametrize the empirical functions are recovered by using optimization procedures to minimize functionals that represent the difference between the solution of the direct problem and the available experimental data. Taking into account that parameter estimation in systems of partial differential equations such as (2.4)–(2.8) is usually unstable (Alvarez 2005; Engl 1993) i.e. the recovered parameters do not depend on the data in a stable way, we include a Tikhonov regularization term when necessary. This regularization enables one to obtain stable approximations of ill-posed inverse problems Tikhonov and Arsenin (1977). The well-posedness of the regularized approximation of the inverse problem of recovering the filtration and permeability reduction functions is proved in Alvarez (2005) in the operator theory framework; a summary is given here in Appendix A.

It is important to notice that the first two equations in the governing system, (2.1) and (2.2), are only weakly coupled to the last Eq. 2.3. Equations 2.1 and 2.2 determine the deposition of the suspended particles, while Eq. 2.3 relates the flow rate U to the pressure drop through the loss of permeability, which depends on the deposited concentration. In fact, in laboratory experiments U is measured, and the decoupling is complete. We are led naturally to split the inverse problem of determining the empirical functions into two separate problems.

First, we recover the filtration function using either the method presented below or the methods presented in Alvarez et al. (2006) and Alvarez (2005). This first inverse problem determines the filtration function from the outlet concentration i.e. the kinetic particle capture rate is calculated from the particle concentration history. Then we recover the permeability reduction function using either the method presented below or the method presented in Alvarez (2005). This second inverse problem determines the formation damage function from pressure drop i.e. the dynamic coefficient that specifies the hydraulic conductivity increase due to particle retention is calculated from the history of the pressure loss on the core.

Remark 3.1 The inverse problem of determining the functions $\lambda(\sigma)$ and $k(\sigma)$ in deep bed filtration is analogous to that of determining the fractional flow function and the total mobility in two-phase transport in porous media. Fractional flow is determined from the fraction of water in the outlet two-phase flux (Welge 1952), while total mobility is determined from pressure drop and injection rate histories (Johnson et al. 1959).

3.1 Optimization procedure

Good algorithms for constrained minimization are the essential tool for the development of efficient methods for general nonlinear programming (Friedlander and Martinez 1994 and Martinez 1998). Such methods apply to problems of the form

$$\text{find } x^* = \arg\min F(x) \text{ subject to } x \in \Omega$$

where $x^*, x \in \mathbb{R}^n$ and n is the number of variables, $F(x) : \mathbb{R}^n \to \mathbb{R}$ is a functional we wish to minimize, and Ω is a convex region in $\mathbb{R}^n$. In essence, these algorithms proceed as follows: at the k-th iteration, the current guess for the minimizer $x^k \in \Omega$ is incremented in the direction of $-\nabla F(x^k)$, yielding $\tilde{x}^k$; the size of the increment is determined from the difference $|F(x^{k-1}) - F(x^k)|$. Next, the resulting point is orthogonally projected on to the feasible set Ω, yielding $\hat{x}^k$. The algorithm may search along the lines from x^k to $\tilde{x}^k$ and $\hat{x}^k$ for better minimizers i.e. linear searches may be conducted before or after the projection. This eventually brings x^k to a local minimizer, either with $\nabla F = 0$ or at the boundary $\partial\Omega$, where the algorithm stops: there is no guarantee the minimizer is unique or global.

This approach is particulary interesting if the projection on to Ω is easy to compute: it is trivial in the case where $\Omega \subset \mathbb{R}^n$ is defined by box boundaries $l_i < u_i$, $i = 1, \ldots, n$. We used an implementation of the spectral projection gradient method with box constraints presented in detail in Birgin et al. (2000), developed by the very authors and very suitable to our problem. The code was designed for large problems (i.e. n large) and is therefore extremely robust for a relatively simple application as the one presented in this work. The number n of parameters to recover does not exceed three in the examples given in the following two sections.

3.2 Recovering the filtration function

In this section we define our functional for solving the first inverse problem: namely, finding the filtration function from the effluent concentration history measured in laboratory experiments.

Before the recovery procedure, a parametrization $\lambda(\sigma; \theta)$ must be chosen for the filtration function, where θ is the set of parameters. The form of these parametric functions and their parameter ranges are dictated by physical properties of the filtration function: concrete examples are discussed in the following two sections. Then we minimize a functional relating the filtration function and the effluent concentration:

$$F^c(\theta, \alpha) = \int_B^A (c(L, t; \theta) - c_{\exp}(t))^2 \mathrm{d}t + \alpha^2 ||\theta - \theta^*||^2 . \tag{3.1}$$

In the first term of the right-hand side, $c_{\exp}(t)$ represents the effluent particle concentration history measured in the laboratory, $c(L, t; \theta)$ is obtained by solving Eqs. 2.4–2.8 for a fixed set of parameters θ, $B = (\phi/U)L$ is the breakthrough time and A is the end time of the experiment. When necessary, we use the second term in the right-hand side, which is a penalization term, where α is the Tikhonov regularization parameter. This term is required to obtain stable solutions only if the parametrization $\lambda(\sigma; \theta)$ has an inadequate form or too many parameters; otherwise, the solution is regularized by the parametrization itself (see Alvarez 2005, p. 146, or Groetsch 1984; Schock 1984).

3.3 Recovering the permeability reduction function

For one-dimensional flow in a rock core, we divide Eq. 2.3 by $1/k(\sigma(x, t))U$, integrate the resulting equation in $[0, L]$ and obtain the following relationship between deposited particle distribution and pressure drop history:

$$-\int_0^L \frac{\mathrm{d}x}{k(\sigma(x, t))} = \frac{k_0}{\mu U} \Delta p(t), \quad 0 \le t \le A. \tag{3.2}$$

As in the previous subsection, first we choose a parametrization $k(\sigma; \beta)$ for the permeability reduction function, with parameter set β. Then we minimize the functional:

$$F^p(\beta, \gamma) = \int_B^A \left(\Delta p(t; \beta) - \Delta p_{\exp}(t)\right)^2 \mathrm{d}t + \gamma^2 ||\beta - \beta^*||^2, \tag{3.3}$$

where $\Delta p(t; \beta)$ is the right-hand side of Eq. 3.2, $\Delta p_{\exp}(t)$ is the experimental data, and γ is the regularization parameter. Once again, the penalization term is required to produce stable solutions in certain cases e.g. when parameterization $k(\sigma; \beta)$ is poorly chosen. Notice that the evaluation of $F^p(\beta, \gamma)$ requires the system of Eqs. 2.4–2.8 to be solved using the filtration function recovered from the effluent concentration history: once we have the deposition of particles $\sigma(x, t)$, then the integrals in Eq. 3.3 can be evaluated numerically.

Remark 3.2 The two inverse problems formulated in this section are consistent with the physical meaning of the unknown functions $\lambda(\sigma)$ and $k(\sigma)$. System (2.1)–(2.2) describes the "kinematics" of deep bed filtration—retention kinetics between suspended and deposited concentrations. The unknown filtration function $\lambda(\sigma)$ is the probability for a particle to be captured during its travel along the unit length i.e. $\lambda(\sigma)$ is a kinematic characteristic of the process, and it is determined from the outlet particle concentration $c(L, t)$.

The unknown permeability reduction function $k(\sigma)$ is a relative rate of particle suspension decrease under constant pressure gradient during the filtration. It is defined from the "momentum balance" Eq. 2.3 and it is therefore a dynamical property of deep bed filtration. It is determined from the histories of pressure drop on the core $\Delta p(t)$ and of flow rate $U(t)$ i.e. from dynamical data.

Granted this consistency, it is important to notice that no specific parametric form was imposed on either λ or k up to this point. The recovery procedure, although developed with this particular problem in view, is mathematically abstract and formally it makes no assumptions about the two functions to be recovered or about the input data. The optimization procedure, however, may impose restrictions: the procedure from Birgin et al. (2000) we chose requires that the functionals have continuous first-derivatives on θ and β, which is the case in the examples given in Sects. 4 and 5.

4 Numerical results with synthetic data

In this section we present an example where the functionals (3.1) and (3.3) are minimized numerically. We use synthetic data $\widetilde{c_e}(t)$ and $\widetilde{\Delta p}(t)$ to calibrate the model and to test the algorithm. We define the parametric form of the filtration and permeability reduction functions as

$$\lambda(\sigma) = \max\{0, \theta_1 - \theta_2\sigma\} \quad \text{and} \quad k(\sigma) = \frac{1}{1+\beta\sigma}, \tag{4.1}$$

which are similar to those typically found in the literature (McDowell-Boyer et al. 1986). We fix $\theta_1 = 1, \theta_2 = 171$ and $\beta = 300$: these are the parameters we wish to recover after solving the inverse problem with the synthetic data. We also assume the inlet concentration $c_i(t)$ to be constant. The synthetic data are constructed in two steps. First, we solve the direct problem for some filtration and permeability damage functions; second, we add random perturbations to this exact solution to simulate observational error inherent to real data.

In order to avoid solving the inverse problem in a space with the same dimension and of the same type as the one in which the functions in (4.1) are defined (see Kaipio and Somersalo 2006 for a discussion on so-called inverse crimes), we fit rational functions $\tilde{\lambda}(\sigma)$ and $\tilde{k}(\sigma)$ to both $\lambda(\sigma)$ and $k(\sigma)$ in (4.1), obtaining

$$\tilde{\lambda}(\sigma) = \frac{1 - 85.2\sigma}{1 + 85.4\sigma + 15{,}737\sigma^2} \quad \text{and} \quad \tilde{k}(\sigma) = \frac{1 - 149\sigma}{1 + 151\sigma - 44{,}640\sigma^2} \tag{4.2}$$

that approximate $\lambda(\sigma)$ and $k(\sigma)$ given in (4.1), but have more parameters. We use the functions (4.2) to solve the direct problem given by the system of Eqs. 2.4–2.8 and the integral Eq. 3.2. Incidentally, the resulting profiles of $\widetilde{c_e}(t)$ and $\widetilde{\Delta p}(t)$ have maximum relative errors of 6.68×10^{-3} and 4.72×10^{-4}, respectively, when compared to the solutions found using the prescribed functions (4.1).

We create four sets of perturbed synthetic data from the one single exact $\widetilde{c_e}(t)$ and $\widetilde{\Delta p}(t)$ solution obtained using $\tilde{\lambda}(\sigma)$ and $\tilde{k}(\sigma)$. More specifically, we introduce random perturbations of the order of 0.01, 0.05, 0.10 and 0.15 i.e. the relative error (noise/signal ratio) of the perturbed data is limited by these given values. The frequency of the added noise is similar to that observed in laboratory experiments (Al-Abduwani 2005). These are shown in Fig. 1.

Solving the inverse problem for each of these four data sets, we recover the parameters for Eq. 4.1. Now we compare the unperturbed synthetic data to the results of the direct problem based on these recovered parameters, and find the relative error to be similar to the one between the unperturbed synthetic data and the solution obtained using the prescribed functions (4.1), as shown in Table 1.

In Fig. 2 the corresponding recovered filtration and permeability reduction functions are shown. We see that these functions are recovered with reasonable accuracy, and smaller relative errors for cleaner data, as expected.

The numerical examples based on synthetic data suggest that the recovery method described here is appropriate for finding the permeability reduction and filtration functions from experimental data.

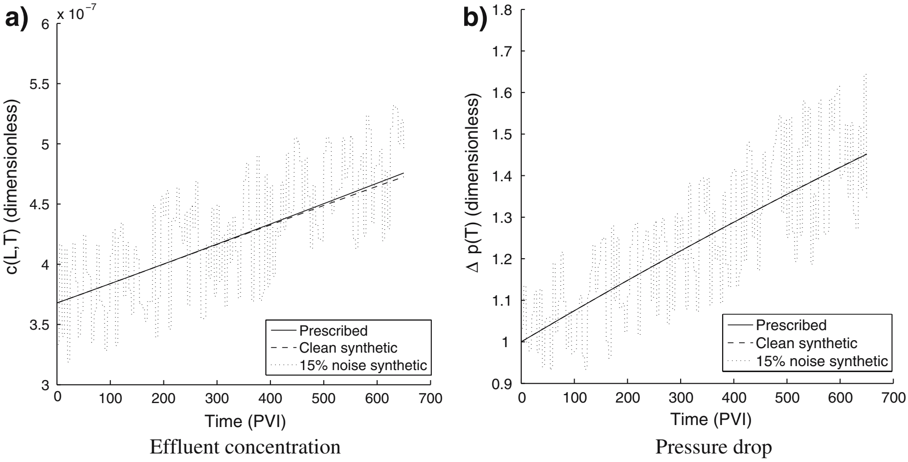


Fig. 1 Synthetic data

Table 1 Relative errors between the unperturbed synthetic data and the solutions obtained using the parameters recovered from the perturbed data sets

Noise	Max error in c_e	Max error in Δp
1%	3.14×10^{-3}	5.21×10^{-4}
5%	5.47×10^{-3}	1.32×10^{-3}
10%	2.09×10^{-2}	1.58×10^{-2}
15%	2.57×10^{-2}	1.70×10^{-2}

Remark 4.1 The optimizations were carried out using the code implemented by the authors of Birgin et al. (2000), after the algorithms detailed therein, which we introduced in Sect. 3. These algorithms implement many complementary minimization strategies and stopping criteria. Many of these criteria are based on values dependent on the scales of the problem, such as the magnitude of the functional being minimized or its gradient vector, and therefore we tuned the optimization constants to the scale of our data. The longest running time we experienced was a little over 1 min on a 2 GHz processor, with 50 evaluations of the cost functional. Since the complexity of the functional is related to the parametrization chosen, we obtain a similar performance when we apply the same procedures to experimental data, as detailed in the next section. This low computational cost is intrinsic to our recovery approach, and one of the advantages of the method.

4.1 Sensitivity analysis

Sensitivity analysis allows one to identify the "best" models, so that the number of parameters is neither insufficient nor is it so large as to render the solution of the parametrized inverse problem not unique. Also, once an optimal set of parameters is obtained, the computation of the corresponding sensitivity matrix allows studying the stability of the proposed inverse method (Sun et al. 2001). Here we perform the sensitivity analysis only for the filtration function given in (4.1).

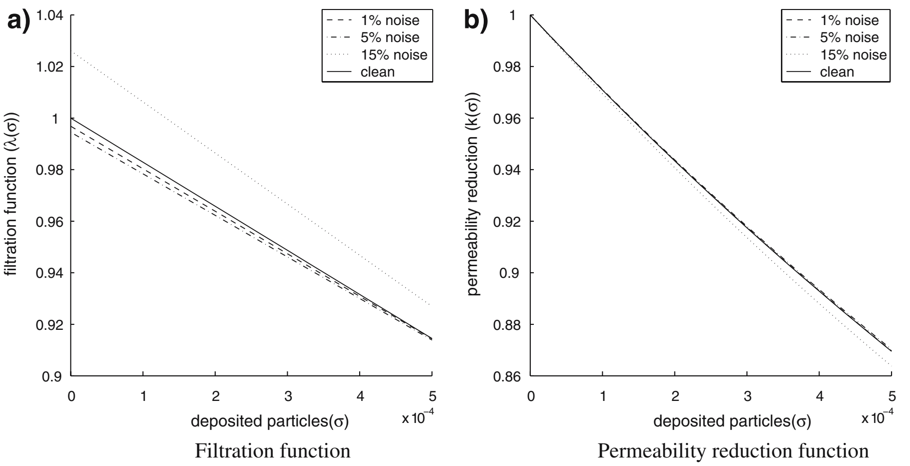


Fig. 2 Empirical functions recovered from noisy synthetic data

To calculate the sensitivity matrix, we use the analytical solution of system (2.1)–(2.2). For simplicity, we transform the physical domain into dimensionless coordinates by the scaling equations

$$X = \frac{x}{L} \quad \text{and} \quad T = \frac{U}{\phi L} t, \tag{4.3}$$

so X is in the [0, 1] range; one unit of dimensionless time T is how long it takes to inject a volume of fluid corresponding to the porous volume of the sample, hence the term *porous volume injected* (PVI). We also define $\sigma(X, T) \equiv \phi^{-1}\sigma(x, t)$ and $\lambda(\sigma(X, T)) \equiv \phi L \lambda(\sigma(x, t))$, so that the Eqs. 2.1–2.2 become

$$\frac{\partial}{\partial T}(c + \sigma) + \frac{\partial c}{\partial X} = 0 \tag{4.4}$$

$$\frac{\partial \sigma}{\partial T} = \lambda(\sigma) c. \tag{4.5}$$

The solution for the system (4.4)–(4.5) with $\lambda(\sigma)$ given by Eq. 4.1, derived in Alvarez (2005), is

$$\sigma(X, T; \theta) = 0, \;\; c(X, T; \theta) = 0 \quad \text{for } T < X, \tag{4.6}$$

$$\sigma(X, T; \theta) = \frac{\theta_1}{\theta_2}\left[1 + \frac{e^{-c_{i0}\theta_2 T}\, e^{(\theta_1 + c_{i0}\theta_2)X}}{1 - e^{-c_{i0}\theta_2 (T - X)}}\right]^{-1} \quad \text{and} \tag{4.7}$$

$$c(X, T; \theta) = \frac{c_{i0}\theta_2 \sigma(X, T)}{\theta_1 (1 - e^{-c_{i0}\theta_2 (T - X)})} \quad \text{for } T > X. \tag{4.8}$$

We wish to study the sensitivity of the recovered parametric function $c(1, T; \theta)$; the coefficients of the sensitivity matrix (see Sun et al. (2001) and Tarantola and Valette (1982)) are defined by

$$S_{\theta_i}(T) = \frac{\partial c(1, T; \theta)}{\partial \theta_i}. \tag{4.9}$$

For a fixed θ, the magnitude of the function $S_{\theta_i}(T)$ reflects the sensitivity of the solution with respect to the i-th parameter at a particular time. The sensitivity coefficients for the filtration function given in Eq. 4.1 are

$$S_{\theta_1} = \frac{-c_{io}\mathrm{e}^{-c_{io}\theta_2(T-1)}\mathrm{e}^{\theta_1}}{\left(1+\mathrm{e}^{-c_{io}\theta_2(T-1)}(\mathrm{e}^{\theta_1}-1)\right)^2}, \quad S_{\theta_2} = \frac{c_{io}^2(T-1)\mathrm{e}^{-c_{io}\theta_2(T-1)}(\mathrm{e}^{\theta_1}-1)}{\left(1+\mathrm{e}^{-c_{io}\theta_2(T-1)}(\mathrm{e}^{\theta_1}-1)\right)^2}. \quad (4.10)$$

From (4.10) we obtain

$$S_{\theta_1}/S_{\theta_2} = -\frac{\mathrm{e}^{\theta_1}}{c_{io}(\mathrm{e}^{\theta_1}-1)(T-1)}. \quad (4.11)$$

Equation 4.11 implies that the sensitivity of $c(1, T; \theta)$ is different relatively to θ_1 and θ_2; as a function of $T > 1$, the solution is relatively more sensitive to θ_1 for smaller values of T. In the particular example we developed in this section, the values of S_{θ_i} are such that $|c(1, T; \theta)/S_{\theta_1}(T)| \approx 1$ and $|c(1, T; \theta)/S_{\theta_2}(T)| \approx 10^3$ in the time interval $1 < T < 650$ i.e. perturbations of θ_1 will reflect on $c(1, T; \theta)$ with the same order of magnitude, whereas the effects of perturbations of θ_2 are reduced by a factor of 10^{-3}. Put simply, this solution is sensitive to the first parameter θ_1, and not to θ_2. These facts discourage the use of more parameters for the filtration function. A two-parameter, non-constant $\lambda(\sigma)$ such as the ones used in this section and in the next, is enough to model most of the experimental data we have analysed. In the last examples in Sect. 5, we use three-parameter filtration functions to account for a different behaviour of the effluent concentration history.

A similar analysis can be conducted for the pressure drop history $\Delta p(T; \theta, \beta)$. However, Δp is a function of $\sigma(X, T; \theta)$ (see Eq. 3.2); therefore, the observations regarding the sensitivity of $c(1, T; \theta)$ are applicable to Δp as well. For the experiment presented in this section, the numerical value of the sensitivity coefficient $\partial(\Delta p)/\partial\beta$ is two orders of magnitude smaller than the actual value of Δp, so the pressure drop is rather insensitive to this single parameter.

5 Numerical results for experimental data

Data from Kuhnen et al.

We applied the method to the effluent particle concentration experimental data described by Kuhnen et al. (2000), shown in Fig. 3. In these experiments, hematite suspensions of equal concentration and varying ionic strengths were injected at an equal rate into sandstone, to investigate the relation between filtration phenomena and electrostatic attraction between oppositely charged particle and porous medium surfaces. No cake formation was observed in these experiments.

In order to apply our empirical model, we tested numerous parametrizations on each of the six data series contained therein, and obtained the following results.

All the data series show an increasing trend. The first three of them go close to the zero filtration rate situation, where $c_e = c_i$ and $\lambda(\sigma) = 0$. For the model to reproduce the first case accurately i.e. to approach $\lambda(\sigma) = 0$ as σ increases, this trend must be taken into account: all successful parametrizations were of the form $\lambda_0 e^{-f(\sigma)}$, where $f(\sigma)$ was a non-negative increasing function. Polynomials of various degrees yielded different results: the best all-around parameterization was $\lambda_0 e^{-\theta\sigma^2}$, for all six data series. For the latter times of the first

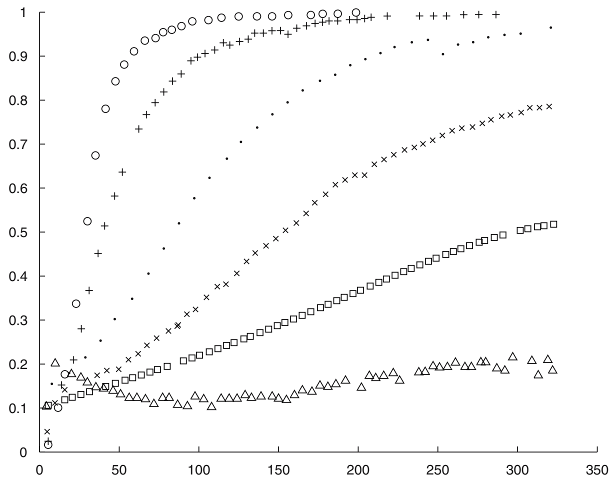


Fig. 3 Effluent concentration histories from Kuhnen et al. (2000), up to 300 PVI. We will refer to these six series by number, from top to bottom

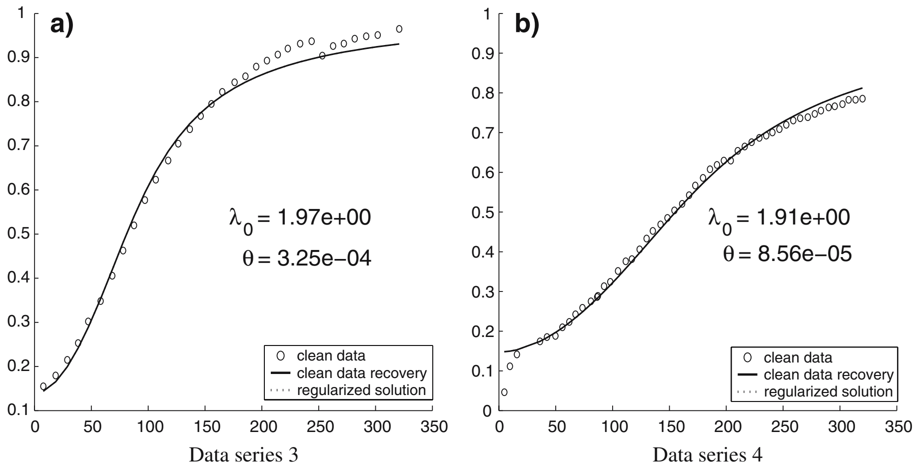


Fig. 4 Best fits for effluent concentration using $\lambda(\sigma) = \lambda_0 e^{-\theta\sigma^2}$

two series, in the late time ranges where $c_e \approx c_i$, a good match was only obtained using $f(\sigma) = \theta\sigma^4$.

We remark that the method proposed in Alvarez et al. (2006) does not work for the zero filtration rate situation observed in the first three data sets, as the quantity actually recovered is $1/\lambda(\sigma)$ rather than $\lambda(\sigma)$. The remaining three do not approach this limit.

Figure 4 shows good matches obtained using the quadratic exponent. Figure 5 shows the difference of the two best parameterizations for series approaching the limit $c_e/c_i \cong 1$.

Restricting the input data to the first 70 PVI of the last four data series, shown as markers in Fig. 6a, we recovered the filtration functions shown in Fig. 6b. We calculated the effluent concentrations using these filtrations and plotted them over the input data in Fig. 6a, with which they coincide visually.

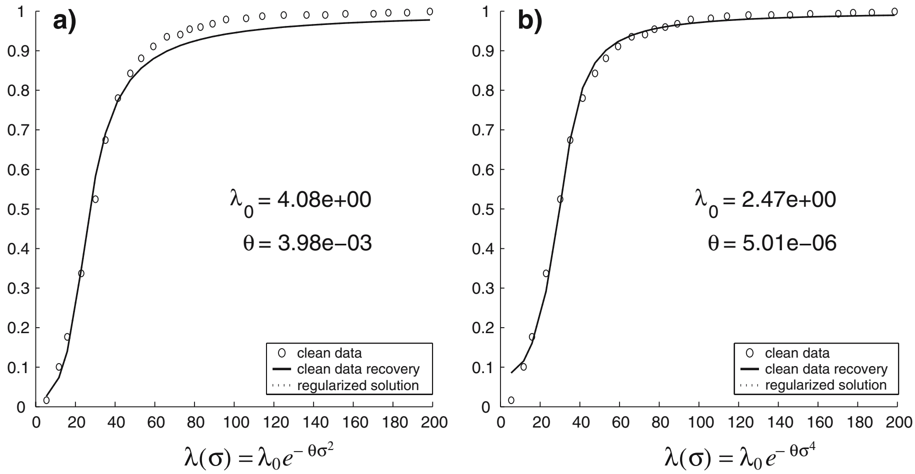
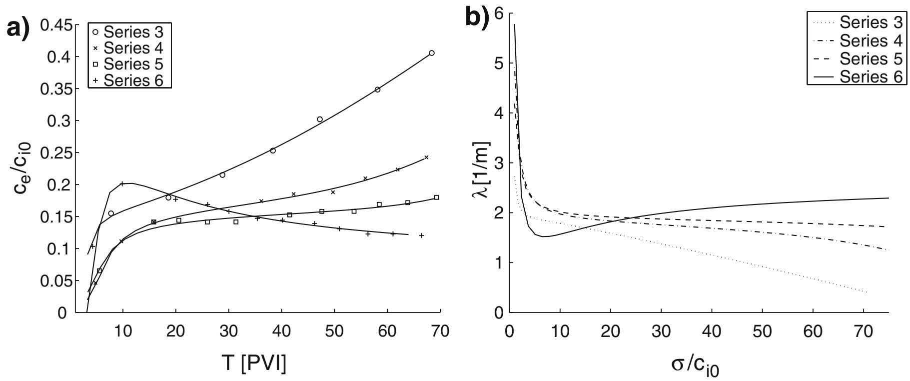


Fig. 5 Profiles recovered using two fits for λ in data series 1



Cropped data and model prediction. Corresponding filtration functions.

Fig. 6 Excellent recovery obtained using the optimization solution shown over smooth approximations of the actual data

Data from Soma and Papadopoulos

Soma et al. performed a series of four experiments injecting oil-in-water emulsions into quartz sand (see Soma and Papadopoulos 1995), using similar conditions and varying the ionic strength of the emulsion, for which they measured both effluent concentration and total permeability reduction i.e. the pressure drop history. There was no cake formation in their experiments.

We applied the empirical model (2.1)–(2.3) to the two experiments with higher ionic strength, where there was enough deposition both to prevent the effluent concentration curve from reaching $c_e/c_i \approx 1$ quickly, and to produce significant permeability reduction.

However, the effluent concentration history curves are not monotone in these two experiments: this is another situation where the method presented in Alvarez et al. (2006) fails to produce good results. The authors Soma and Papadopoulos (1995) attribute this lack of

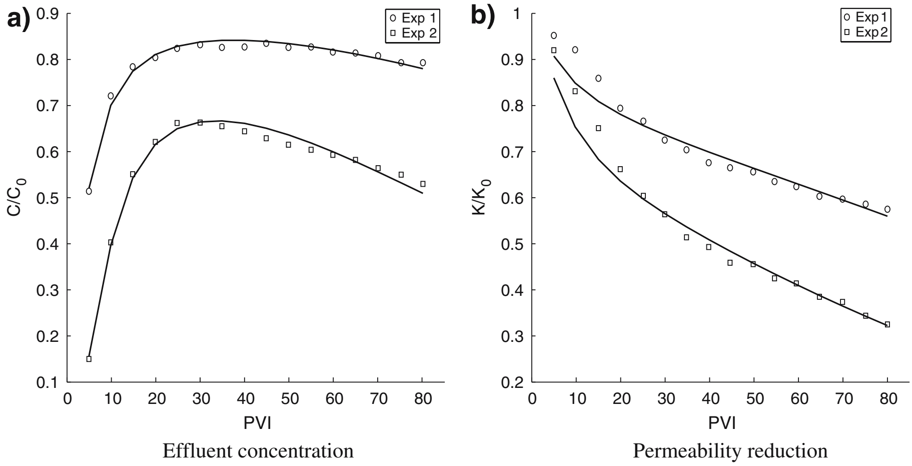


Fig. 7 Recovered histories and original data in two experiments

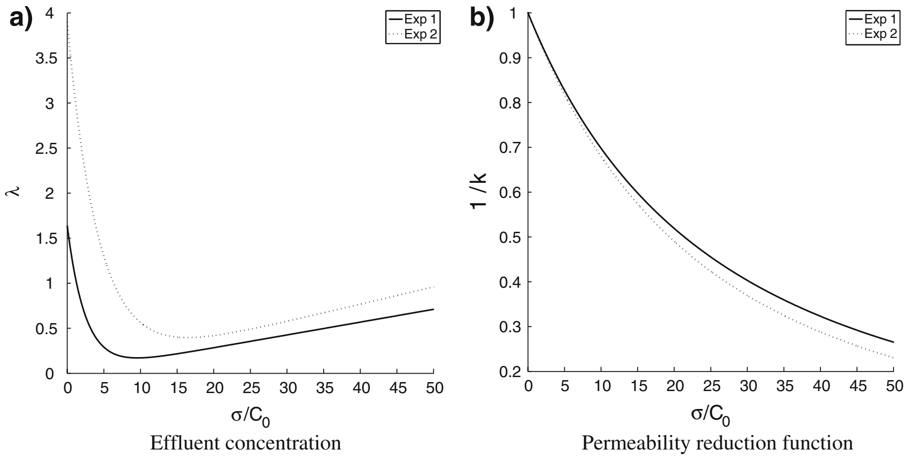


Fig. 8 Recovered empirical functions in two experiments

monotonicity to a greater attraction of oil droplets in the suspension to previously deposited oil rather than to the bare pore surface. In terms of the empirical $\lambda(\sigma)$ coefficient, this translates into a non-monotone behaviour of the filtration function, one that increases after a certain value of σ is reached. To account for this behaviour, we chose the parametrization

$$\lambda(\sigma) = \theta_1(e^{-\theta_2\sigma^2} + \theta_3\sigma); \quad \theta_1, \theta_2, \theta_3 > 0. \tag{4.12}$$

For the permeability reduction function, we use the inverse of a second-degree polynomial, which is compatible with the literature (McDowell-Boyer et al. 1986) and suitable for our calculations. Higher degree polynomials did not yield better results.

For both data sets, the model reproduced well both the effluent concentration and permeability reduction curves, as shown in Fig. 7. The parametric empirical functions recovered solving the inverse problems are shown in Fig. 8.

6 Conclusion

The recovery method described here consists of solving two inverse problems: in the first one the filtration function is determined from the effluent concentration evolution, and in the second one the formation damage function is determined from the pressure drop history. In the cases examined, the method yields good matches between synthetic/experimental and predicted data.

The results given in Sects. 4 and 5 indicate that the method is robust and flexible, for it allows the analysis of experimental data that the other recovery methods in the literature cannot analyze. The method proposed in Alvarez et al. (2006) requires that $\lambda(\sigma)$ does not approach zero i.e. it cannot deal with data where c_e/c_i approaches one. The three-point method presented in Bedrikovetsky et al. (2001) and (2003) assumes $\lambda(\sigma)$ to be a constant λ_0, which is equivalent to assuming that c_e/c_i is constant in time after breakthrough: this is not the behaviour observed in any of the experiments presented in Sect. 5. The distinguishing feature of the method presented in this work is that it imposes few restrictions on the form of the functions to be recovered, providing a flexible tool for studying experimental data. At the same time, it is easier to implement than other methods equally adequate for practical purposes such as the method for recovering the permeability reduction developed in Alvarez (2005). It is also flexible in the sense that, by accommodating different physical conditions under a minimal set of macroscopic parameters, all built into the recovered permeability damage and filtration functions, it can be readily applied to variations of the model (2.1)–(2.3) which cannot be solved numerically by (2.7)–(2.8).

One must keep in mind, however, that the method assumes that both functions to be recovered can be represented by simple parametric expressions, and that the phenomena conforms to all modelling assumptions made in Herzig et al. (1970) and the other works derived from there, based on which this inverse problem was formulated and solved. As this happens to be the case in many real industrial applications, the method proposed here is viable for determining the filtration and formation damage functions from laboratory corefloods for use in predicting the injectivity decline, formation damage and contaminant propagation in a variety of petroleum and environmental engineering projects.

We did not fully automate the method: at this stage, part of the recovery procedure is a skillful choice of parametrizations, which requires good understanding of the physics related to the experimental data. Full automation would require analyzing dozens of varied cases. This is not warranted yet, as the model (2.1)–(2.3) may not be valid for all filtration problems.

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Appendix A: Well-posedness of the regularized inverse filtration problem

Mathematical problems that are supposed to be solved using real data containing errors must be well-posed i.e. $\mathcal{O}(1)$ perturbations cannot arise in the answer from infinitesimal perturbations of the data. In this section we prove that the inverse problem formulated in Eq. 3.1 is well-posed in the sense of Tikhonov (see Lavrent'ev and Savel'ev 1991). To do so, a feasible subset of parameters is chosen, such that the minimization problem has a unique minimum, and that small perturbations of experimental data produce small parameter variations. We rewrite the inverse problem in the framework of operator theory. Several results on regularization of non-linear operators are used to prove the well-posedness of the inverse problem (see Engl et al. 2000).

The stability and convergence results obtained here are based on Binder et al. (1994); Engl et al. (2000); Kravaris and Seinfeld (1985) and Colonius and Kunisch (1989). We choose

$$D(G^c) = \{\lambda \in H^2[0,1], \quad \text{such that} \quad \lambda > \bar{\lambda}_1\}, \tag{A.1}$$

with $\bar{\lambda}_1$ constant. Here, $H^2[0,1]$ is the Hilbert space of functions defined in the interval [0,1] with norm

$$\|f\|^2 = \int_0^1 \left(|f(x)|^2 + |f'(x)|^2 + |f''(x)|^2\right) \mathrm{d}x.$$

Let us define the non-linear operator

$$G^c : D(G^c) \subset H^2[0,1] \to L^2[0,A], \quad G^c(\lambda) = c(1,\cdot;\lambda), \tag{A.2}$$

where λ represents the filtration function and $c(1,\cdot;\lambda)$ is the effluent concentration obtained from the solution of the system (2.5), (2.7)–(2.8). Now $L^2[0,1]$ is the Hilbert space of functions defined in the interval [0,1] with norm

$$\|f\|^2 = \int_0^1 |f(x)|^2 \mathrm{d}x.$$

From the well-posedness of the direct problem (Alvarez 2005), for each λ there exists a unique function $c(1,\cdot;\lambda)$, so the operator in (A.2) is well-defined. Notice that the domain $D(G^c)$ is closed and convex, therefore it is weakly closed. Let us define

$$\mathcal{M} = \{\lambda \in H^2[0,1] \quad \text{such that} \quad \bar{\lambda}_2 < \lambda < \bar{\lambda}_3\}, \tag{A.3}$$

where $\bar{\lambda}_2$ and $\bar{\lambda}_3$ are constants.

We recall the definition of $H^s[0,1]$ when s is not integer: Let a_n be the Fourier coefficient of a function in [0,1], namely

$$a_n = \int_0^1 \mathrm{e}^{-2\pi n x i} f(x)\, \mathrm{d}x.$$

The norm in $H^s[0,1]$ is

$$\|f\| = \left(\sum_{n=-\infty}^{\infty} \left(1 + |n|^{2s}\right) |a_n|^2\right)^{1/2}.$$

We have following:

Theorem A.1 *Let λ represent the filtration function and $c(1,\cdot;\lambda)$ the solution of the system (2.1)–(2.2), with initial and boundary condition given in (2.4) and (2.5). Let $\eta \geq 0$ and consider the domain*

$$D_\eta(G^c) = \{\lambda \in H^{2+\eta}[0,1] \quad \textit{such that} \quad \lambda > \bar{\lambda}_2\}. \tag{A.4}$$

The following assertions are valid.

(i) The (non-linear) operator

$$G^c : D(G^c) \subset H^{2+\eta}[0,1] \to L^2[0,A], \quad G^c(\lambda) = c(1,\cdot;\lambda), \tag{A.5}$$

is continuous and injective.

(ii) Let $D(G^c)$ be as in Eq. A.1. Then the operator in (A.2) is weakly closed and compact.
(iii) The map $G^c : \mathcal{M} \to G^c(\mathcal{M})$ is continuous and has continuous inverse.

Proof (i) Let $\lambda_n \to \lambda$ in $H^{2+\eta}[0, 1]$ with $\eta \geq 0$. Since $H^{2+\eta}$ is compactly embedded in $C^1[0, 1]$ then $\lambda_n \to \lambda$ uniformly in $C^1[0, 1]$, from Remark 2.3 it follows that $G^c(\lambda_n) \to G^c(\lambda)$ in $L^2[0, A]$. The injectivity is a consequence of the uniqueness of the solution of the system of equations (2.1)–(2.5) and (2.6).

(ii) Let $\{\lambda_n\}$ be a sequence in $D(G^c)$ converging weakly in $H^2[0, 1]$ towards λ. Since $D(G^c)$ is weakly closed, then $\lambda \in D(G^c)$ and since $H^2[0, 1]$ is compactly embedded in $C^1[0, 1]$, which is the Banach space of functions with continuous derivatives and norm $\|f\| = \max_{[0,1]} |f(x)| + \max_{[0,1]} |f'(x)|$, then $\lambda_n \to \lambda$ in $C^1[0, 1]$. By (i) $G^c(\lambda_n) \to G^c(\lambda)$ in $L^2[0, A]$. Thus, G^c is compact, hence weakly closed.

(iii) Notice that G^c is continuous in $\mathcal{M}$ by (ii). Moreover, $\mathcal{M}$ is a compact subset of $C[0, 1]$, because it consists of uniformly bounded functions in $H^1[0, 1]$ (Cullum 1971). Therefore (iii) is a consequence of the Lemma of Tikhonov. □

From (ii) in Theorem A.1 and Proposition 10.1 in Engl et al. (2000), the inverse problem of determining the filtration function λ in $G^c(\lambda) = b$ with given $b = c_e(\cdot)$ is locally an ill-posed problem. Hence, the Regularization of Tikhonov is used to find stable solutions. The regularized solution is determined as the minimizer over $D(G^c)$ of the functional

$$\lambda \to ||G^c(\lambda) - c_e(\cdot)||^2_{L^2[0,A]} + \alpha^2 ||\lambda - \lambda^*||^2_{H^1[0,1]}, \tag{A.6}$$

where α is the regularization parameter. Since G^c is weakly closed, stability and convergence of the regularized solution follow from Theorem 10.2 and 10.3 in Engl et al. (2000).

Let us denote by θ a parametrization of the filtration function $\lambda(\sigma)$. Neglecting the interaction between the parameters i.e. by assuming that they are uncorrelated, we obtain

$$||\lambda - \lambda^*||^2_{H^1[0,1]} \approx ||\theta - \theta^*||^2 \tag{A.7}$$

where $||\cdot||$ denotes some appropriate norm in the parameter space. Thus, from Eq. A.7, we see that the penalization functional can be written in terms of the parameters θ.

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